

# Decentralized Multi-Client Functional Encryption

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Introduction

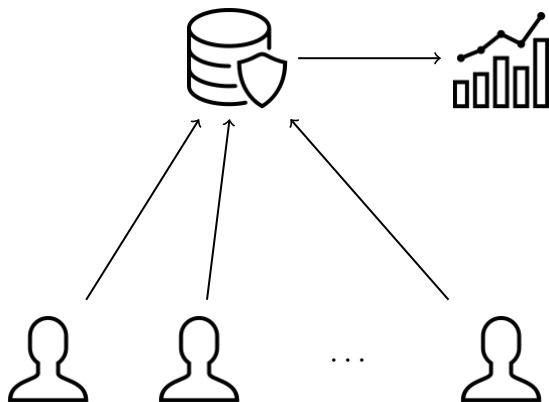
Definitions

Centralized scheme

Decentralized scheme

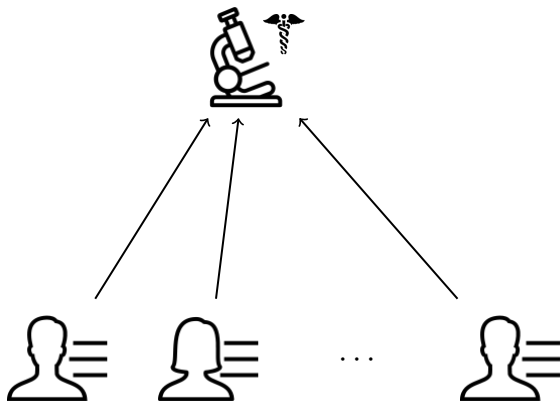
# Context

## Privacy and data analysis



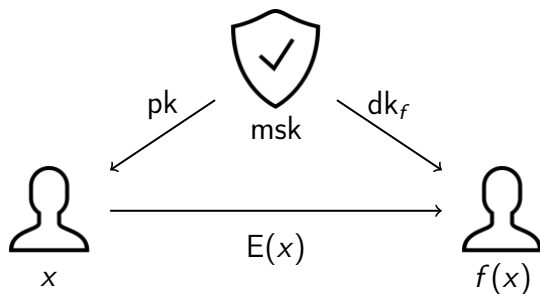
# Context

Example: medical studies



# Context

Functional Encryption (FE) [BSW11]:



IP-FE [ABDP15]:  $f(\vec{x}) = \langle \vec{x}, \vec{y} \rangle$  for a given  $\vec{y}$

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# Description

Involved parties:

Trusted authority



Receiver



Senders

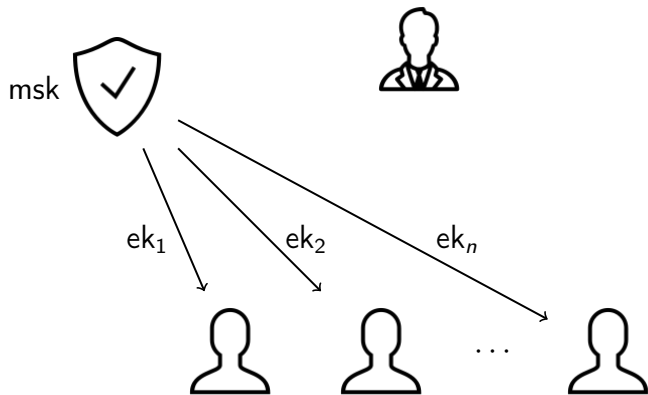


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## Description

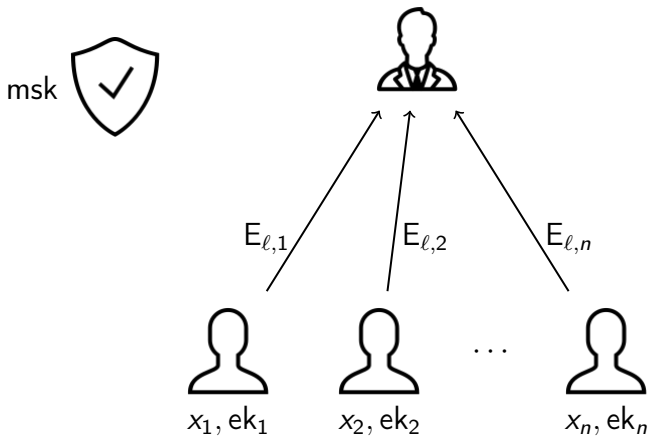
Setup: Sets  $msk$ , returns personal encryption keys  $ek_i$





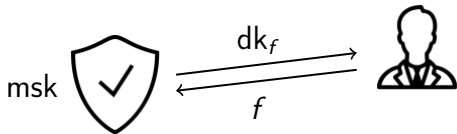
# Description

Encrypt: Returns encrypted value  $E_{\ell,i}$



## Description

DKeyGen: Given a function  $f$ , returns a decryption key  $dk_f$



# Description

Decrypt: Returns the value  $f(\vec{x})$



$f(\vec{x})$



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# Security model

Malicious adversary

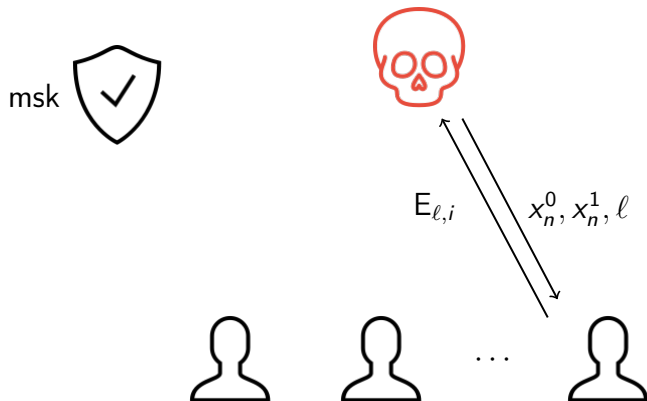


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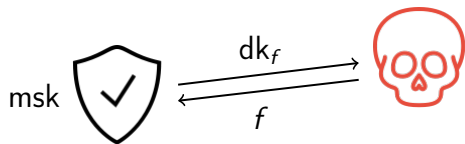
# Security model

Encryption queries:  $\text{QEncrypt}(i, (x_i^0, x_i^1), \ell)$



# Security model

Decryption key queries:  $\text{QDKeyGen}(f)$



# Security model

Corruption of senders:  $Q_{\text{Corrupt}}(i)$



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# Security model

Conditions of trivial win:

- ▶  $f(\vec{x}^0) \neq f(\vec{x}^1)$
- ▶  $x_i^0 \neq x_i^1$  for any corrupted sender  $i$



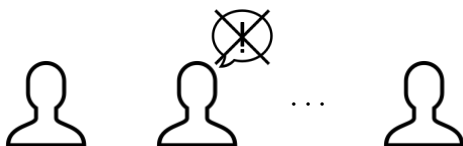
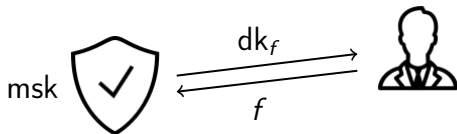
# Security model

Variants:

- ▶ selective security
- ▶ static corruption

# Why decentralization

No right management for senders



# Why decentralization

Corruption of the authority



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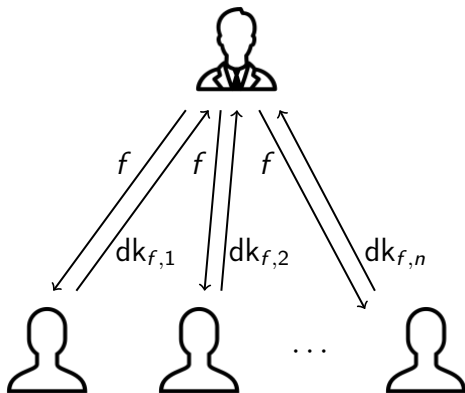
# Why decentralization

Requires decentralization



## Impact on the definition

DKeyGenShare: returns a part  $dk_{f,i}$  of the future key  $dk_f$



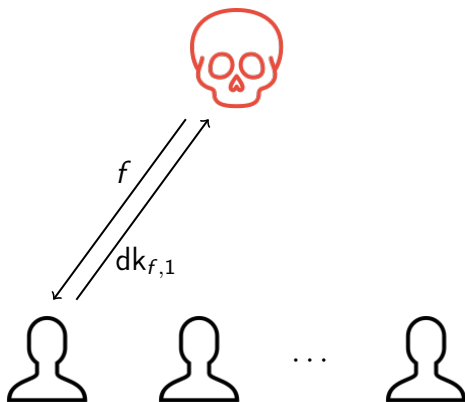
## Impact on the definition

DKeyGenComb: Build the decryption key  $dk_f$  from  $(dk_{f,i})_i$



# Impact on the security

Partial key queries:  $\text{QDKeyGen}(f, i)$



Introduction

Definitions

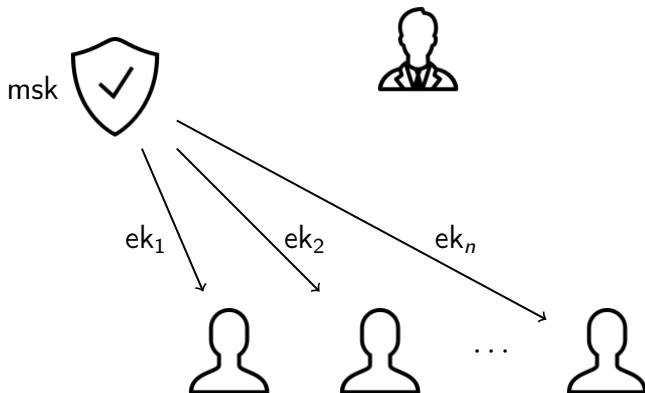
Centralized scheme

Decentralized scheme



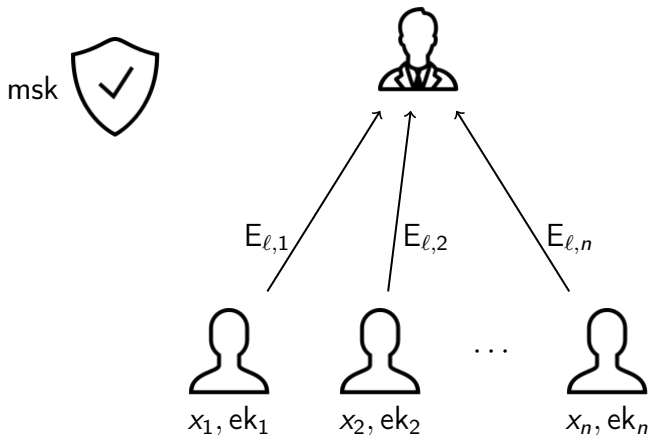
# Description

Setup( $\lambda$ ):  $ek_i \leftarrow \mathbb{Z}_p$ ,  $msk = (ek_i)_i$ ,  $pp = (\mathbb{G}, \mathcal{H} \text{ onto } \mathbb{G})$



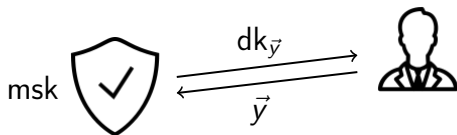
# Description

Encrypt( $ek_i, x_i, \ell$ ):  $E_{\ell,i} = \mathcal{H}(\ell)^{ek_i} \cdot g^{x_i} \in \mathbb{G}$



## Description

$\text{DKeyGen}(\text{msk}, \vec{y}): dk_{\vec{y}} = \langle \text{msk}, \vec{y} \rangle = \langle (ek_i)_i, \vec{y} \rangle \in \mathbb{Z}_p$



# Description

$$\text{Decrypt}(dk_{\vec{y}}, \ell, \vec{E}_\ell): \prod_i E_{\ell,i}^{y_i} \cdot \mathcal{H}(\ell)^{-dk_{\vec{y}}}$$



# Description

Given  $E_{\ell,i} = \mathcal{H}(\ell)^{ek_i} \cdot g^{x_i}$  and  $dk_{\vec{y}} = \langle \text{msk}, \vec{y} \rangle$ , the detail of the correctness is:

$$\begin{aligned}\alpha &= \prod_i E_{\ell,i}^{y_i} \cdot \mathcal{H}(\ell)^{-dk_{\vec{y}}} \\ &= \prod_i (\mathcal{H}(\ell)^{ek_i} \cdot g^{x_i})^{y_i} \cdot \mathcal{H}(\ell)^{-\langle \text{msk}, \vec{y} \rangle} \\ &= \mathcal{H}(\ell)^{\sum_i ek_i y_i} \cdot g^{\sum_i x_i y_i} \cdot \mathcal{H}(\ell)^{-\langle \text{msk}, \vec{y} \rangle} \\ &= g^{\langle \vec{x}, \vec{y} \rangle}\end{aligned}$$

Then solve the discrete logarithm of  $\alpha$

# Security

- ▶ adaptive ciphertext queries
- ▶ adaptive corruption queries

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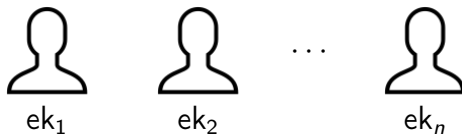
Decentralized scheme

# Idea

Problem: interactions for DKeyGen



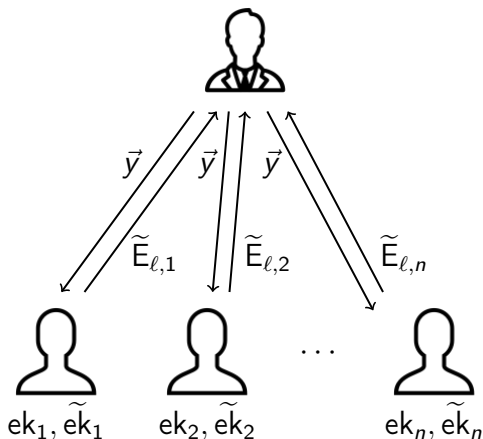
MPC between senders





# Idea

Solution:  $\widetilde{\text{MCFE}}$  to build  $dk_{\vec{y}} = \langle (ek_i y_i)_i, \vec{1} \rangle$



# Idea

Specific problem: key as group element instead of scalar



# Idea

Specific solution: pairings

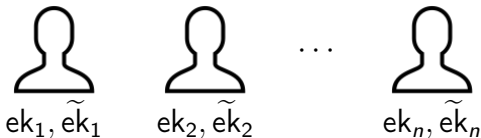
- ▶ message related MCFE in  $\mathbb{G}_1$
- ▶ key related  $\widetilde{\text{MCFE}}$  in  $\mathbb{G}_2$

## Description

Setup( $\lambda$ ):  $ek_i, \tilde{ek}_i \leftarrow \mathbb{Z}_p, sk_i = (ek_i, \tilde{ek}_i),$   
 $pp = (\mathbb{G}_1, \mathcal{H}_1, \mathbb{G}_2, \mathcal{H}_2, \mathbb{G}_T, e, \tilde{dk} = \sum_i \tilde{ek}_i)$

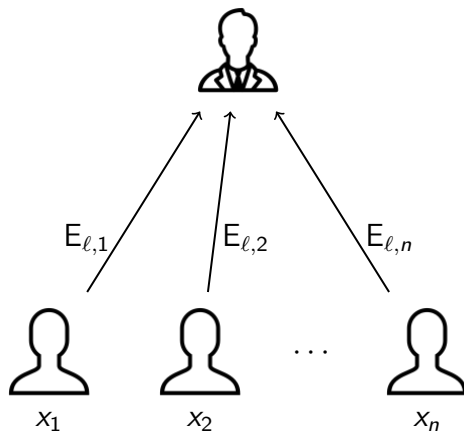


MPC between senders



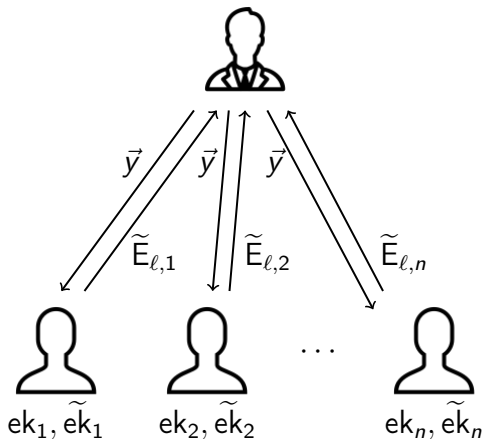
## Description

Encrypt( $ek_i, x_i, \ell$ ):  $E_{\ell,i} = \mathcal{H}_1(\ell)^{ek_i} \cdot g_1^{x_i} \in \mathbb{G}_1$



## Description

$\text{DKeyGenShare}(sk_i, \vec{y}): \tilde{E}_{\ell, i} = \mathcal{H}_2(\vec{y})^{\tilde{ek}_i} \cdot g_2^{\text{ek}_i y_i} \in \mathbb{G}_2$



## Description

$$\begin{aligned} \text{DKeyGenComb}((\tilde{E}_{\ell,i})_i, \vec{y}): dk_{\vec{y}} &= \sum_i dk_{\vec{y},i} \cdot \mathcal{H}_2(\vec{y})^{-\tilde{dk}} \\ &= g_2^{\sum_i ek_i y_i} \in \mathbb{G}_2 \end{aligned}$$



## Description

$$\text{Decrypt}(\text{dk}_{\vec{y}}, \ell, \vec{E}_\ell): e(\prod_i E_{\ell,i}^{y_i}, g_2) \cdot e(\mathcal{H}_1(\ell), -\text{dk}_{\vec{y}})$$



$$g_T^{(\vec{x}, \vec{y})}$$



...





# Security

- ▶ adaptive ciphertext queries
- ▶ static corruption queries

# Conclusion

In this paper we saw:

- ▶ MCFE primitive and scheme
- ▶ Security supporting corruptions
- ▶ Decentralization

Open questions:

- ▶ Security
- ▶ Number size
- ▶ Other function families

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- ▶ Number size
- ▶ Other function families

Thank you !